33.10 A viscous liquid with a specific gravity of 1.1 is pumped with a volume flow rate of $100gpm$. The suction pressure measured at the inlet of the pump is $5psi$ of vacuum. The discharge pressure measured 3 feet above the pump outlet is $10psig$. The diameter of the piping reduces from 4 inches on the suction side to 2 inches on the discharge side. The pump has a mechanical efficiency of 80%. What is the input power to the pump?

A. $3/4hp$

B. $1hp$

C. $1^1/4hp$

D. $1^1/2hp$

A useful and common form of the Bernoulli Equation can be applied for pumping applications such as this. A term is added to the original Bernoulli Equation, h_A, to reflect the head pressure added by the pump. Isolate h_A and collect like terms.

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + h_A = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f$$

$$h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f$$

First consider the difference in static pressure. The suction pressure measured at the pump inlet is $5psi$ of vacuum. A vacuum pressure is implied to be gauge pressure as it is not possible to pull a vacuum more than atmospheric pressure. The discharge pressure is $10psig$. Ignore the 3ft elevation for the time being; this will be accounted for when analyzing the Δz term. Note the specific weight of the viscous liquid is 1.1 times the specific weight of water. See Specific Gravity in the reference handbook. Convert square inches to square feet to ultimately obtain units of ft (of viscous liquid).

$$\frac{P_2 - P_1}{\gamma} = \frac{(10psi - (-5psi))\left(\frac{144in^2}{1ft^2}\right)}{\left(62.4\frac{lb_f}{ft^3}\right)(1.1)} = 31.47ft$$

Although the velocity term is often ignored, in this case the discharge piping diameter is reduced with respect to the suction side piping; therefore the velocity will increase for a given volume flow rate. To be safe, include the velocity term in the solution. Use the Steel Pipe Friction Tables to obtain the actual diameter and velocity where possible.

$$D_1 = 4in\,(nominal)$$

$$Q_1 = 100gpm$$

$$v_1 = 2.52\frac{ft}{s}$$

Since the table in the reference handbook stops at $50gpm$ for a nominal $2in$ diameter pipe, the velocity must be calculated using $Q = vA$. A rule of thumb that may be used for converting volume flow rates is $450\frac{gpm}{\frac{ft^3}{s}}$.

$$D_2 = 2.067in$$

$$Q_2 = 100gpm$$

$$v_2 = \frac{Q_2}{A_2} = \frac{\left(100\frac{gal}{min}\right)}{\left(450\frac{gpm}{ft^3/s}\right)\left(\frac{\pi}{4}\right)\left(\frac{2.067in}{12\frac{in}{ft}}\right)} = 9.54\frac{ft}{s}$$

$$\frac{v_2^2 - v_1^2}{2g} = \frac{\left(9.54\frac{ft}{s}\right)^2 - \left(2.52\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 1.31ft$$

For completeness, write the difference in height term as well.

$$\Delta z = z_2 - z_1 = 3ft$$

Although the problem does not state explicitly, it is reasonable to ignore the friction losses because the measurements are taken close to the suction and discharge of the pump, and no equivalent length or fitting information has been given. Determine h_A:

$$h_A = 31.47ft + 1.31ft + 3ft = 35.8ft$$

The input power to the pump is the brake horsepower, i.e. Brake HP, which can be expressed as the water horsepower, i.e. *whp*, divided by the pump efficiency, η, where whp is a function of the volume flow rate and head added by the pump.

$$bhp = \frac{whp}{\eta_p}$$

$$whp = \frac{Q\Delta h \cdot SG}{3960}$$

Combine the two equations, substitute, and solve. Note that h_A has units of ft of a column of viscous liquid with a specific gravity, $SG > 1$. Therefore, it is appropriate to include the specific gravity to account for the increased energy required to pump a viscous liquid as compared with water ($SG_{water} = 1$).

$$bhp = \frac{Q_{[gpm]}\Delta h_{[ft]} \cdot SG}{3960 \cdot \eta_p} = \frac{(100)(35.8)(1.1)}{(3960)(.8)} = 1.24hp$$

Answer C